# E-ProTran: Efficient Probabilistic Transformers for Forecasting

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Can we design probabilistic transformers with higher efficiency?
Yes, **E-ProTran** has improved inference speed and scalability with comparable performance!

## Motivation and Contributions

**Transformers** excel in handling long-range dependencies in sequential data and show promise in time series analysis. However, **their complexity often results in overparameterization, extended training times, and scalability challenges**, especially with complicated generative model assumptions. In this work, we propose Efficient Probabilistic Transformers (E-ProTran), a re-design of Probabilistic Transformers (ProTran) to mitigate these problems.

## Formulation and Problem Setting

We work with multivariate time series $x_{1:T} \in \mathbb{R}^{T\times d}$, where $x_t \in \mathbb{R}^d$ is a $d$-dimensional data vector at the discrete time step $t \in \mathbb{N}^+$. We parameterize a generative model with joint of $p_{\psi,\theta}(x_{1:T}, z_{1:T}|x_{1:t_0}, c_{1:T})$ can be used for forecasting in temporal settings. The baseline work ProTran uses

$$p_\psi(z_{1:T}|x_{1:t_0}, c_{1:T}) = \prod_{l=1}^{L}\prod_{t=1}^{T} p_\psi(z_t^{(l)}|z_{1:t-1}^{(l)}, z_{1:T}^{(l-1)}, x_{1:t_0}, c_{1:T}) \quad (1)$$

$$p_\theta(x_{1:T}|z_{1:T}) = \prod_{t=1}^{T} p_\theta(x_t|z_t^{(L)}) \quad (2)$$

where $L$ is number of layers in the encoder, $T$ is length of the temporal data.

## E-ProTran

For our model, we have the generative model with components

$$p_\psi(z_{1:T}|x_{1:t_0}, c_{1:T}) = \prod_{t=1}^{T} p_\psi(z_t|x_{1:t_0}, c_{1:T}) \quad (3)$$

$$p_\theta(x_{1:T}|z_{1:T}) = \prod_{t=1}^{T} p_\theta(x_t|z_t). \quad (4)$$

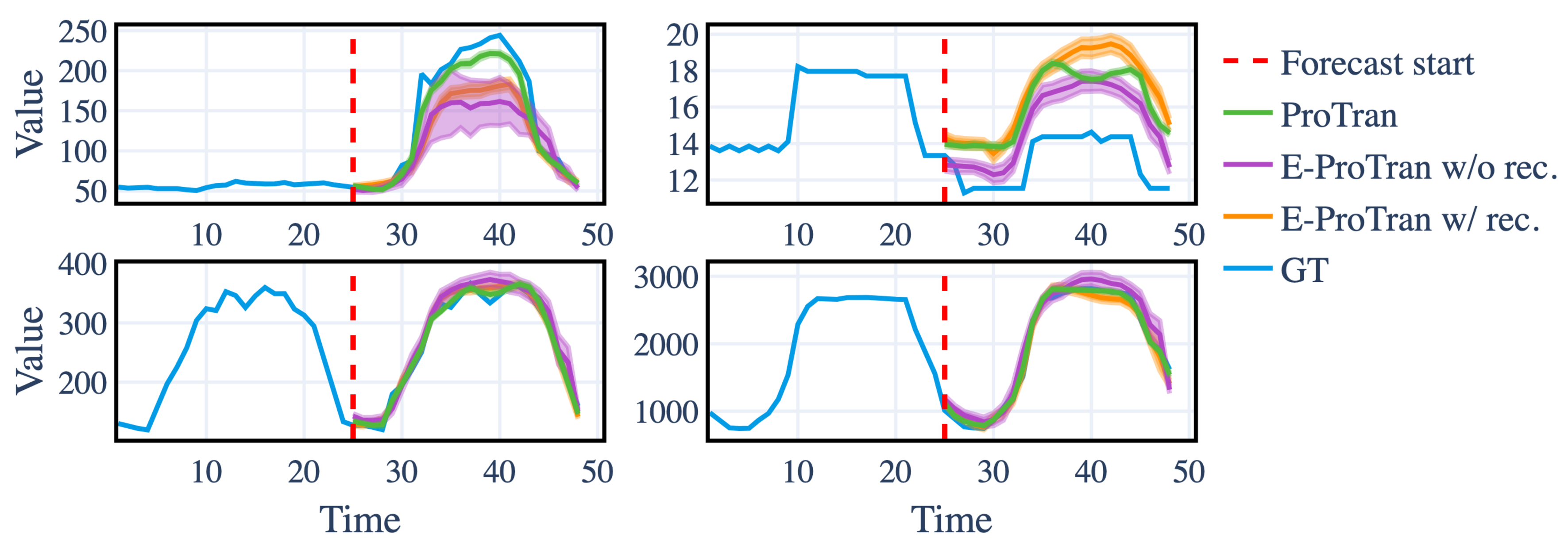
We opt to use

- Non-autoregressive attention to reduce computational overhead while allowing information flow over time through layer attention.
- Causal attention to lower layers to minimize propagation of error from later time steps, and make sure our predictions are independent of the forecasting length.
- Stochastic $z_t$ only at the latent bottleneck; otherwise, it introduces noise in the forward pass and thus affects the gradients during training.

## Results

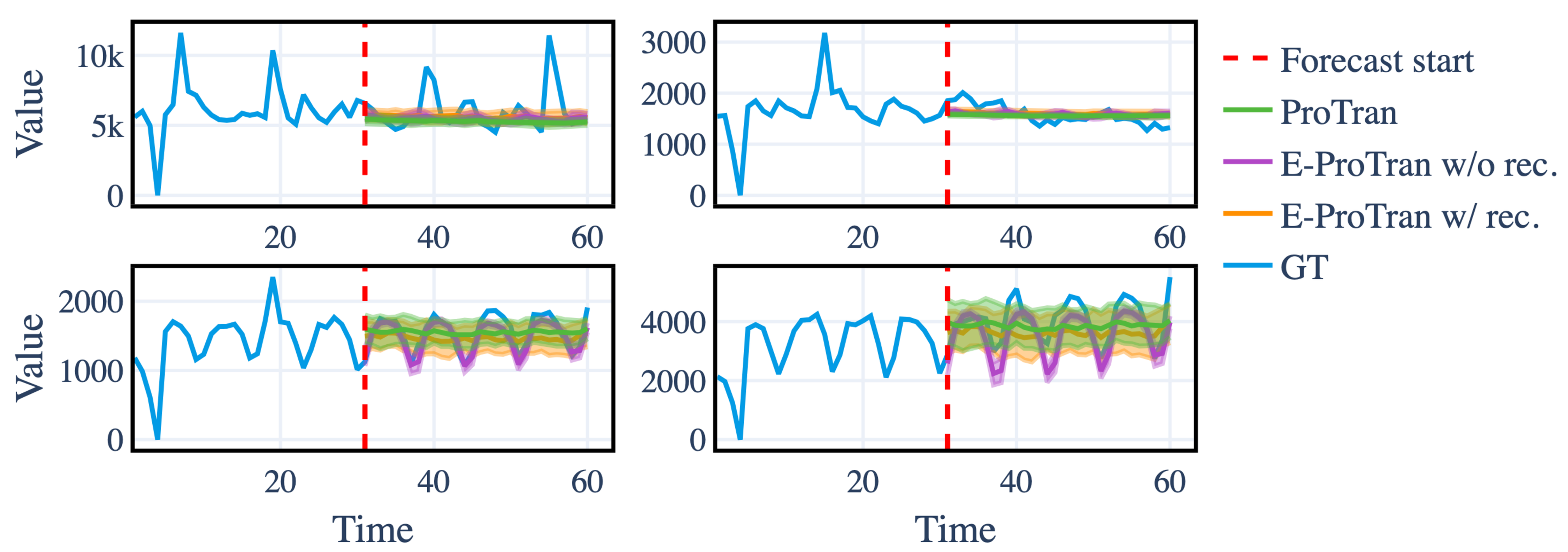
We present the performance metrics for Electricity and Wikipedia. The average predictions are from 100 forward passes; shaded areas show 1 standard deviation.

| Model | | | Electricity | | | | |
|---|---|---|---|---|---|---|---|
| Type | Att. | Rec. | CRPS$_{\text{sum}}$ | CRPS | RMSE | #Params | Forw. Pass (sec) |
| ProTran | L I A | ✓ | 0.030 | **0.069** | **439.2** | 501,924 | 0.064 ± 0.001 |
| E-ProTran | L I A | ✓ | **0.024** | 0.075 | 501.7 | 382,084 | 0.044 ± 0.001 |
| E-ProTran | L I | ✓ | – | – | – | – | – |
| E-ProTran | L | ✓ | **0.024** | 0.072 | 530.2 | **315,652** | **0.002 ± 0.000** |
| E-ProTran | L I A | ✗ | 0.030 | 0.077 | 582.8 | 382,084 | 0.044 ± 0.001 |
| E-ProTran | L I | ✗ | – | – | – | – | – |
| E-ProTran | L | ✗ | 0.029 | 0.079 | 598.7 | **315,652** | **0.003 ± 0.001** |




**Forecasting on Electricity** for the first test sequence. We show 4 of 370 dims.

| Model | | | Wikipedia | | | | |
|---|---|---|---|---|---|---|---|
| Type | Att. | Rec. | CRPS$_{\text{sum}}$ | CRPS | RMSE | #Params | Forw. Pass (sec) |
| ProTran | L I A | ✓ | 0.066 | 0.320 | 5909.2 | 1,522,080 | 0.187 ± 0.003 |
| E-ProTran | L I A | ✓ | 0.075 | 0.353 | 6020.0 | 1,259,584 | 0.141 ± 0.005 |
| E-ProTran | L I | ✓ | 0.063 | 0.328 | 5932.2 | 1,126,720 | **0.007 ± 0.001** |
| E-ProTran | L | ✓ | 0.063 | **0.311** | 5936.8 | **1,060,416** | **0.007 ± 0.002** |
| E-ProTran | L I A | ✗ | 0.081 | 0.354 | 5983.3 | 1,259,584 | 0.139 ± 0.005 |
| E-ProTran | L I | ✗ | 0.054 | 0.327 | 5945.1 | 1,126,720 | **0.007 ± 0.002** |
| E-ProTran | L | ✗ | **0.053** | 0.316 | **5906.7** | **1,060,416** | **0.007 ± 0.002** |




**Forecasting on Wikipedia** for the first test sequence. We show 4 of 2000 dims.

## Looking for More?

* Equal contribution.

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